Date
-03/08/2020

B.Sc (Physics) Part-II, Paper-IV, Group-A

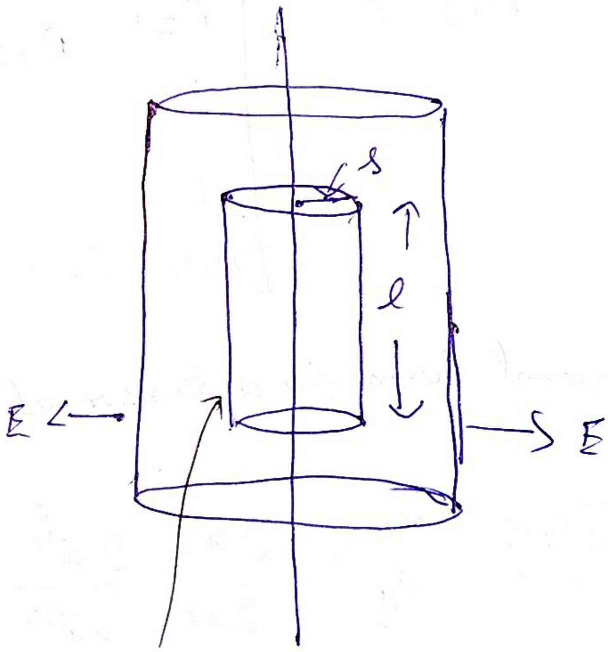
sunil.phy30@gmail.com

Problems on Gauss's law:

(Q1) A long cylinder carries a charge density that is proportional to the distance from the axis: $\rho = ks$, where k is some constant. Find the electric field inside this cylinder.

Soln.

See fig., The cylinder having length l and radius s is Gaussian cylinder



Gaussian surface, cylinder length l and radius s.

Now Gauss's law

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$= \frac{1}{\epsilon_0} \int \rho \, d\tau = \frac{1}{\epsilon_0} \int (ks') s' ds' d\phi dz$$

$d\tau = s' ds' d\phi dz$ {for cylindrical coordinate}

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{k}{\epsilon_0}\int_0^s s'^2 ds' \int_0^{2\pi} d\phi \int_0^l dz = \frac{2\pi k l}{\epsilon_0}\int_0^s s'^2 ds'$$

$$= \frac{2\pi k l s^3}{3\epsilon_0} \quad \text{——} \ (1)$$

Whereas $\int |\vec{E}|\, da = |\vec{E}| \int da = |\vec{E}|\cdot 2\pi s l$ —— (2)

From (1) & (2)

$\vec{E}$ is directed radially outward.

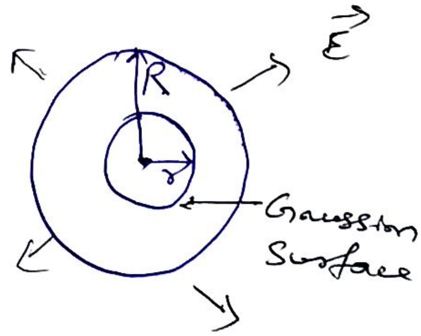
$$|\vec{E}|\, 2\pi s l = \frac{2\pi k l s^3}{3\epsilon_0}$$

or $|\vec{E}| = \frac{1}{3\epsilon_0} k s^2$

$$\boxed{\vec{E} = \frac{1}{3\epsilon_0} k s^2 \hat{s}}$$

(Q.2) Use Gauss's law to find the electric field inside a uniformly charged sphere (charge density ρ).

Soln. As in above problem, we have drawn the Gaussian surface. Here the surface will be spherical.



Using Gauss's law, $\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_0} Q_{enc}$

$$Q_{enc} = \int \rho\, d\tau = \rho \int r'^2 \sin\theta\, dr'\, d\theta\, d\phi$$

$$Q_{enc} = \rho \int_0^r r'^2 dr' \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi = \rho\left(\frac{r^3}{3}\right) 2\cdot 2\pi = \frac{4}{3}\pi r^3 \cdot \rho$$

$$\therefore \int \vec{E}\cdot d\vec{a} = |\vec{E}|\, 4\pi r^2 = \frac{1}{\epsilon_0}\frac{4}{3}\pi r^3 \rho \Rightarrow \boxed{|\vec{E}| = \frac{1}{3\epsilon_0}\rho r}$$

or $\boxed{\vec{E} = \frac{1}{3\epsilon_0}\rho r \hat{r}}$

Since total charge 'Q' is given by $Q = \frac{4}{3}\pi R^3 \rho \Rightarrow \rho = \frac{3Q}{4\pi R^3}$

Now $\vec{E} = \frac{1}{3\epsilon_0}\left(\frac{3Q}{4\pi R^3}\right) r\, \hat{r} \Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}\vec{r}}$ Ans